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UTILITIES ESTIMATED FROM ACTUAL DECISIONS IN READINESS MEASUREMENT

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Key Words: Readiness, Utilities, Risk and Uncertainty

INTRODUCTION

Some of the rationales for using utility functions to measure commanders' attitudes towards risk and uncertainty as an aid in the readiness measurement problem were developed in Technical Report No. 12. Interview approaches have been used in most studies which have been conducted of attitudes towards risk and uncertainty, and an interview approach using mission scenarios and responses of the commander to choice questions in the specified circumstances was presented in the aforementioned report. The scenarios are an attempt to reflect real circumstances and the utility functions resulting from this procedure can be informative and useful.

However, these interview procedures can be somewhat impeded by the fact that the commander does not have to bear the consequences of his readiness evaluation regardless of the realism of the scenario. For this reason, some people have suggested that decisions made under real operating conditions may be more revealing of a person's attitude towards risk and uncertainty.

The purpose of this report is to develop models which can improve the readiness measurement system by focusing on the use of the commander's actual readiness evaluations of his ship to measure his attitudes towards risk and uncertainty. It is concerned with readiness for individual missions as well as the overall readiness of the unit.

The evaluation process is viewed both from a conceptual and an empirical point of view. A commander's evaluation of his ship is based on objective data regarding materiel, personnel, maintenance, etc. as well as the commander's knowledge, experience, and attitude towards risk and uncertainty (his utilities).

We present in this report some conceptual models of readiness measurement or evaluation which have the potential for improving planning, communication, control, and prediction. These models are not intended to be descriptions of how commanders rate a ship's readiness or of how they should rate the ship. Rather these models are developed to reveal the relationships among the relevant elements in the evaluation process and to provide a framework for empirically determining a commander's utility function from his actual readiness evaluations.

We then consider some approaches for using these models for estimating utilities and for predicting ship performance.

GENERAL MODEL

We assume in our general model that the ship may be used for various missions, which are denoted 1,2,...,m. The probabilities that the ship will be called upon to perform each mission are denoted $q_1, q_2, \ldots, q_m$ ($q_j \geq 0$; $\sum_j q_j = 1$).

Performance on missions is affected by various factors such as materiel, personnel, maintenance, etc. These are denoted 1,2,...,f. The relative importance of each factor for each mission may vary and is denoted V_{ij}, the relative importance of factor i for mission j ($V_{ij} \geq 0$; $\sum_i V_{ij} = 1$ for $j = 1,2,\ldots,m$).

The commander is assumed to evaluate the readiness of factor i to perform in mission j. This evaluation is denoted R_{ij}.

This conceptual model can be represented by the following array:

$$
\begin{array}{ccc}
 & & \underline{\text{Missions}} \\
 & & \begin{array}{ccccccc} 1 & 2 & 3 & \cdots & j & \cdots & m \\ q_1 & q_2 & q_3 & \cdots & q_j & \cdots & q_m \end{array} \\
\underline{\text{Factors}} & \begin{array}{c} 1 \\ 2 \\ \cdot \\ \cdot \\ i \\ \cdot \\ \cdot \\ \cdot \\ f \end{array} &
\left[\begin{array}{ccccccc}
 & & & & \vdots & & \\
 & & & & \vdots & & \\
 & & & & \vdots & & \\
 & & & & \vdots & & \\
\cdots & \cdots & \cdots & \cdots & (V_{ij};R_{ij}) & & \\
 & & & & & & \\
 & & & & & & \\
 & & & & & & \\
 & & & & & &
\end{array}\right]
\end{array}
$$

The estimate of the overall readiness of the ship can be made using these elements in a number of different ways. Two of the possible ways for estimating overall readiness, denoted R^*, are:

1. Expected value method

$$R^* = \sum_i \sum_j V_{ij} q_j R_{ij}$$

2. Minimum rating or weakest link method

$$R^* = \underset{(i,j)}{\text{Minimum}} \; R_{ij}$$

SPECIAL CASE OF ONE FACTOR AND ONE MISSION

We will next consider in some detail the readiness evaluation, R_{ij}, for one particular factor i and one particular mission j.

Let the possible values of R_{ij} be $1,2,\ldots,R$. It is assumed that there is an actual state of readiness* or level of performance which is obtained from people other than the commander.

The various possible actual states of readiness or levels of performance are labeled $1,2,\ldots,S$. In the traditional decision theory framework, these states of readiness or performance levels are called states of nature.

R is usually equal to S, although it need not be.

In making his readiness evaluation, the commander chooses a readiness rating, r, from $1,2,\ldots,R$. The consequences or

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*In practice, the actual state of readiness is presumably estimated by evaluators based upon performance (in maneuvers or in combat) or by higher-level officers who review the factors affecting the readiness of the ship. These officially determined "actual" states of readiness are, of course, affected by the utilities or attitudes towards risk and uncertainty of the evaluators and reviewers.

effects of this choice on the commander depend on, among other things, the actual state of readiness (one of 1,2,...,S).

How does the commander choose an r? In this model we postulate that his observations, knowledge, and experience provide one aspect of his decision and his attitude towards risk and uncertainty provides a second aspect. He makes various observations on his ship and, based on his knowledge and previous experience, estimates the probabilities of the different possible actual states of readiness.* These probability estimates, which are not necessarily made consciously, are denoted $p_1, p_2, \ldots, p_S$ ($p_s \geq 0$; $\sum_s p_s = 1$). His attitude towards risk and uncertainty are summarized by his utilities, U_{ij}, which denote the utility of readiness rating r when the actual rating is s.

The situation is represented in the following array:

Commander's Readiness Rating

$$
\begin{array}{lcc}
 & & \begin{array}{ccccccc} 1 & 2 & \cdots & r & \cdots & R \end{array} \\
\text{Actual State of Readiness (State of Nature)} &
\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ \\ p_S \end{array} &
\begin{array}{c} 1 \\ 2 \\ \\ s \\ S \end{array}
\left[\begin{array}{cccccc}
 & & & \vdots & & \\
 & & & \vdots & & \\
 & & & \vdots & & \\
\cdots & \cdots & \cdots & U_{sr} & & \\
 & & & & &
\end{array} \right]
\end{array}
$$

*It could be argued that the commander's attitude towards risk and uncertainty might also exert some influence on his estimates of these probabilities, but this effect should be of secondary importance.

The rating that is used can now be computed by choosing that rating r which will give the largest value of $\sum_{s=1}^{S} p_s U_{sr}$.

Some important statistical questions relating to this model are:

1. Given past experience with a commander's readiness ratings and the actual states of readiness on n evaluation occasions $(r_1,s_1),(r_2,s_2),\ldots,(r_n,s_n)$, how can U_{sr} and perhaps the p's be estimated?

2. Given past experience on n occasions and a commander's readiness rating r_{n+1}, for the $n+1^{th}$ evaluation occasion, how should the actual state of readiness, s_{n+1}, be predicted? In other words, the problem is to predict the actual state of readiness from past experience with a commander and his readiness rating.

AGGREGATED MODEL

In this section, a less detailed model is presented. Here the different missions and different factors are not explicitly stated. (They are, of course, considered by the commander in making his readiness evaluations.) The possible overall readiness ratings of the ship are labeled $r = 1,2,\ldots,R$. The actual performance ratings are given by $s = 1,2,\ldots,S$. As in the previous model, the commander estimates the probabilities of the actual states of readiness by $p_1,p_2,\ldots,p_S$ from his observations on the ship and his knowledge and experience.

His utilities are denoted by W_{sr} in this aggregated model. The model can again be presented in an array:

Commander's Readiness Rating

Actual State of Readiness (State of Nature)

$$
\begin{array}{ll}
 & \begin{array}{ccccc} 1 & 2\ \ldots & r & \ldots & R \end{array} \\
\begin{array}{cc} p_1 & 1 \\ p_2 & 2 \\ \vdots & s \\ p_S & S \end{array} & \left[\begin{array}{ccccc} & & \vdots & & \\ & & \vdots & & \\ \cdots & \cdots & W_{sr} & & \\ & & & & \end{array}\right]
\end{array}
$$

The difference between the model and the one in the previous section is that all the factors and missions are aggregated here. The model postulates that the commander chooses the readiness rating r which maximizes $\sum_s p_s W_{sr}$.

The same kinds of statistical questions arise relating to this model as to the previous disaggregated one: How can the commander's utilities, W_{sr}, and perhaps the p's, be estimated? How can actual states of readiness be predicted from the commander's readiness rating?

The further development of these types of models could be helpful in understanding the dynamics of the readiness measurement process so that, for example, we would know the sensitivity of the system to errors in the different parameters and variables. This could be valuable for designing improvements of the procedures. For example, it is possible that time and effort could be

spent on collection of data which could yield relatively larger effect on the final readiness measures than the current allocation provides.

ILLUSTRATIONS OF APPROACHES AND PROBLEMS

To illustrate the approaches and problems, we will now discuss methods for estimation and prediction using the aggregated model as our example. We will use $R = S = 2$. There are two states of nature (actual states of readiness): $s = 1$ (satisfactory) and $s = 2$ (unsatisfactory) as well as two ratings for the commander of the ship, $r = 1$ (satisfactory) and $r = z$ (unsatisfactory).* The situation is represented by a square array:

*At the present time, the Navy uses a system for its C-ratings in which $R = S = 4$. 1 indicates the most ready condition and 4 indicates the least ready. We have been told that a large proportion of all commander ratings are either 2 or 3, with ratings of 1 and 4 being used relatively infrequently. If one wanted to apply the current ratings to our simplified example, C - 1 and C - 2 ratings could be considered satisfactory ($r = 1$) and C - 3 and C - 4 could be considered unsatisfactory ($r = 2$). (Or, depending on the actual usage of the various C-ratings, C - 1, C - 2, and C - 3 could be considered satisfactory ($r = 1$) and only C - 4 be considered unsatisfactory.)

$$
\begin{array}{llll}
 & & & \text{Commander's Readiness Rating (r)} \\
 & & & \begin{array}{cc} 1 & 2 \end{array} \\
\text{Actual State of Readiness} & P & 1 & \multirow{2}{*}{$\begin{bmatrix} 0 & C_{12} \\ C_{21} & 0 \end{bmatrix}$} \\
\text{(State of Nature)} & 1-P & 2 &
\end{array}
$$

The values of C_{21} and C_{12} are the numerical assessments of the seriousness of situations $(s = 2, r = 1)$ and $(s = 1, r = 2)$ respectively. When $s = r$, a zero cost is assumed. These costs can be thought of as negative utilities.

As previously discussed, the commander's choice of P depends on a combination of his knowledge and experience with his observations of the ship and his choice of values for C_{21} and C_{12} depends on his utilities.

The commander's procedure for choosing a rating is to pick $r = 1$ when $(1 - P)C_{21} \leq PC_{12}$ and $r = 2$ otherwise. The previous inequality is equivalent to choosing $r = 1$ when

$P \geq \dfrac{C_{21}}{C_{12} + C_{21}}$. $\dfrac{C_{21}}{C_{12} + C_{21}}$ will be denoted by c.

Thus, the commander rates the ship as satisfactory $(r = 1)$ when P is greater than a critical value c. c depends only on the ratio of utilities, C_{12}/C_{22}, since

$$c = \frac{C_{21}}{C_{12} + C_{21}} = \frac{1}{1 + (C_{12}/C_{21})} .$$

The commander's rating depends on P and c, with P incorporating (not necessarily consciously) observations evaluated by his knowledge and experience while c is indicative of attitudes towards risk and uncertainty.

To estimate c and make predictions about s from rating r, various circumstances relating to degrees of information about P can exist. At one extreme is the unlikely condition when the commander's P is known. A more likely case, requiring more investigation, occurs when the commander's assessment of P is related to the observations of the ship. Finally, a very useful and reasonable model results when something is known about the distribution of P.

In the usual decision theory situation, there is a known distribution of observations for each state of nature. Based on these distributions, the a priori probability for each state of nature is revised to a new a posteriori probability for each state after each new observation or group of observations. This situation is not exactly present in our case because the relationships between the observations and the states of nature (actual states of readiness) are very complicated and are not sufficiently known or defined.

<u>P's and r's Known For n Occasions:</u>

In this situation, the data for n occasions consists of $(P_1,r_1),(P_2,r_2),\ldots,(P_n,r_n)$. Since for $r=2$, $P<c$ and for $r=1$ $P \geq c$, we obtain bounds $\underline{P} \leq c \leq \bar{P}$, where $\underline{P}$ is the maximum of the P's associated with $r=2$ and $\bar{P}$ is the minimum of the P's associated with $r=1$. The bounds $\underline{P} \leq c \leq \bar{P}$ can also be translated

to give bounds for W_{12}/W_{22} as $\frac{1-\bar{P}}{\bar{P}} \leq \frac{W_{12}}{W_{22}} \leq \frac{1-\underline{P}}{\bar{P}}$. This case has mathematical features similar to those in the revealed preference approach in economic theory.

<u>Distribution of P Known</u>:

In a number of circumstances it is reasonable to assume that some knowledge of the distribution of P is available. This information can sometimes be obtained by statistical studies of the actual ratings of a number of ships on a number of maneuvers. Suppose the probability density function of P is denoted by f(p). It is also indicated later that any assumption about f(p) can be tested statistically by means of suitable data.

Possible forms of f(p) are the uniform distribution, various kinds of triangular distributions and the Beta distribution family indexed by two parameters.

The probability that the commander rates the ship as satisfactory $(r = 1)$ is

$$\text{Prob } (r = 1) = \int_c^1 f(p)dp \ .$$

The probability of $r = 1$ can be estimated by the fraction, g, of satisfactory ratings on n occasions. When function f(p) is known, c can then be estimated by $\hat{c}$ from

$$g = \int_{\hat{c}}^1 f(p)dp \ .$$

For example, when f(p) is the uniform distribution, $f(p) = 1$ for

$(0 \leq p \leq 1)$ then

$$g = 1 - \hat{c} \qquad \text{giving} \qquad \hat{c} = 1 - g \ .$$

For the triangular distribution $f(p) = 2(1 - p)$ the resulting estimate c c is obtained from

$$g = \int_{\hat{c}}^{1} 2(1 - p)dp = (1 - \hat{c})^2$$

giving estimate $\hat{c} = 1 - \sqrt{g}$.

<u>c Known</u>:

When c is also known, this model can be developed to study how the probability that the actual performance is satisfactory $(s = 1)$ relates to the commander's ratings.

Without the commander's rating,

$$\text{Prob}(s = 1) = \int_{o}^{1} \text{Prob}(s = 1 | P = p) f(p)dp$$

$$= \int_{o}^{1} pf(p)dp = E(P) \ .$$

With the commander's rating, say, $r = 1$, the conditional probability is,

$$\text{Prob}(s = 1 | r = 1) = \text{Prob}(s = 1 | P > c)$$

$$= \frac{\int_{c}^{1} pf(p)dp}{\int_{c}^{1} f(p)dp} = E(P | P > c)$$

To illustrate, when $f(p)$ is the uniform distribution,

$$\text{Prob}(s = 1) = E(P) = \frac{1}{2}$$

and,

$$\text{Prob}(s = 1 | r = 1) = E(P | P \geq c) = \frac{1}{2}(1+c)$$

$$\text{Prob}(s = 1 | r = 2) = E(P | P < c) = \frac{1}{2}c \quad .$$

When $f(p)$ and c are known, the joint distribution of s and r can be derived. For instance, again using the uniform distribution as an illustration, the joint and marginal distributions are

s \ r	1	2	Marginal
1	$\frac{1}{2}(1 - c^2)$	$\frac{1}{2}c^2$	$\frac{1}{2}$
2	$1 - c - \frac{1}{2}(1 - c^2)$	$c - \frac{1}{2}c^2$	$\frac{1}{2}$
Marginal	$1 - c$	c	1

From the joint distribution, correlations and conditional distributions can be calculated. For example,

$$\text{Prob}(s = 1 | r = 1) = \frac{\text{Prob}(s = 1, r = 1)}{\text{Prob}(r = 1)} = \frac{\frac{1}{2}(1 - c^2)}{1 - c} = \frac{1}{2}(1 + c)$$

Assumptions regarding f(p) and c can be tested statistically from data of commander's readiness ratings and of actual states of readiness on n occasions $(r_1,s_1),(r_2,s_2),\ldots,(r_n,s_n)$. For example, $\mathrm{Prob}(s=1, r=1)$ is estimated by the fraction of the n occasions in which $s=1$ and $r=1$.

The model can also be used to predict actual performance using the commander's ratings and to study how good these predictions can be expected to be.

The various models can, of course, be studied when more than two rating levels are used, giving rise to multivariate distributions.

In addition, if the ratings of different factors and missions are incorporated, as is done in our general model, it is very likely that better prediction procedures can be developed.